

Find the equation of the tangent line of this equation at point (1,1)

$$X^2 + XY + Y^2 = 3$$

SOLUTION

STEP 1

You can use a very simple logic to determine the value of X and the value of Y and the logic is, since the combined components of the equation is equal to 3, we can mathematically conclude that the value of each of the three components is 1 because $1 + 1 + 1 = 3$

$$X^2 = 1 \quad XY = 1 \quad Y^2 = 1$$

Therefore $X = 1 \quad Y = 1$ because $XY = 1$

$$X^2 + 1 + Y^2 = 3 \quad X^2 + Y^2 = 3 - 1$$

$$= X^2 + Y^2 = 2$$

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STEP 2

Transform the equation to satisfy the equation of a circle this will help determine the point of origin of the line (1,1) and also determine the radius.

$$X^2 + 1 + Y^2 = 3 \qquad X^2 + Y^2 = 3 - 1$$

$$\text{So } X^2 + Y^2 = 2$$

This implies that the point of origin is at (0,0) and the radius is simply the $\sqrt{2}$ So the radius is 1.41

Now we can determine the slope of the line from (0,0) to (1,1) then determine the slope of the tangent line and the equation of the tangent line at (1,1)

$$\text{The slope} = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{1 - 0}{1 - 0} = \frac{1}{1} = 1$$

So the slope is 1

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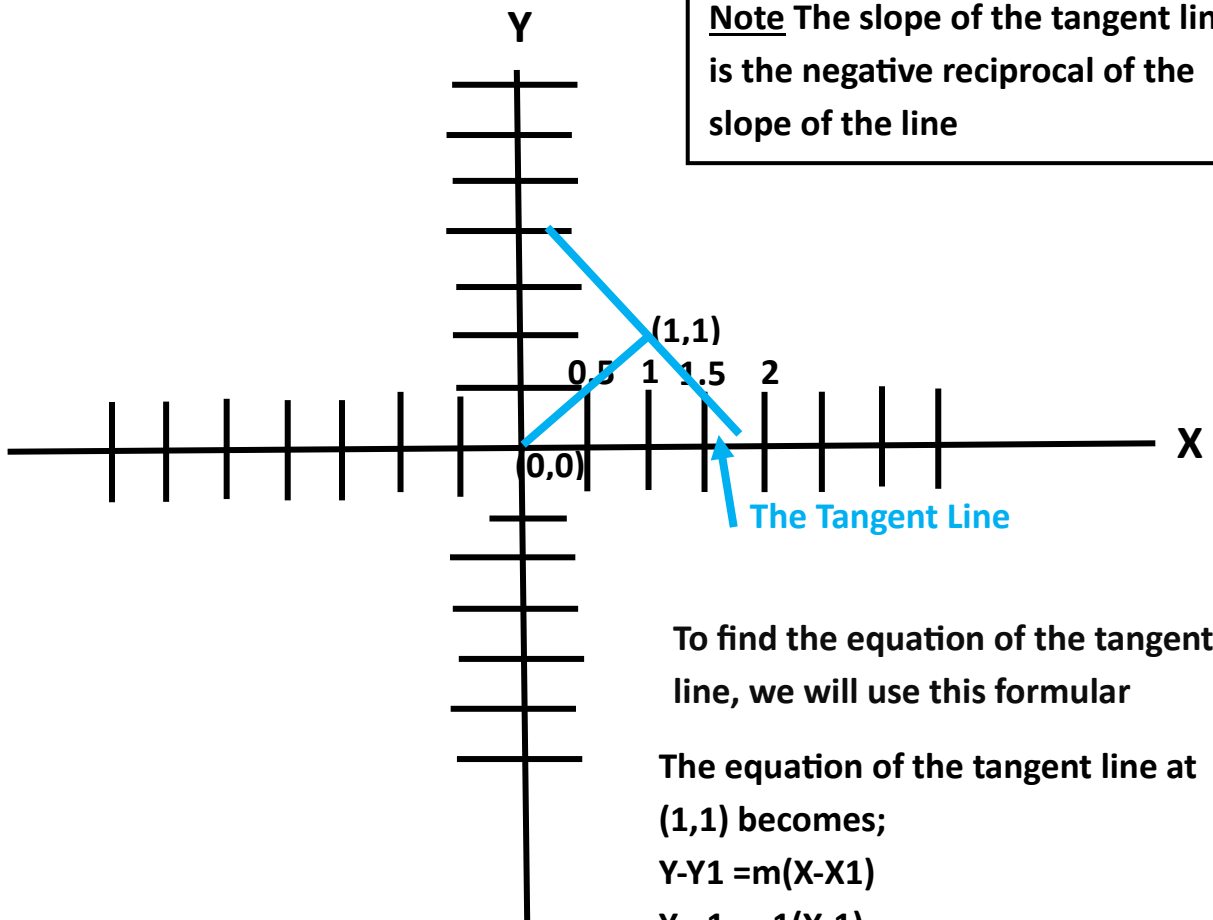
STEP 3

Now since the slope is 1 and the slope of the tangent line is the negative reciprocal of the slope, we can conclude that the slope of the tangent line is -1

So the slope of the tangent line is -1

To find the equation of the tangent line, we find the slope of the tangent line at point (1,1)

Note The slope of the tangent line is the negative reciprocal of the slope of the line



To find the equation of the tangent line, we will use this formular

The equation of the tangent line at (1,1) becomes;

$$Y - Y_1 = m(X - X_1)$$

$$Y - 1 = -1(X - 1)$$

$$Y - 1 = -X + 1$$

$$Y = -X + 1 + 1$$

$$Y = -X + 2$$

$$\underline{Y = -X + 2}$$