Using implicit differentiation to find the equation of a tangent line at a particular point.

Question

Find the equation of the tangent line of this equation at point (1,1)

$$X^3 Y^4 = 4X^2 Y + 1$$

Solution

Step 1

Find the derivative of the equation $X^3 Y^4 = 4X^2 Y + 1$

$$3X^2 Y^4 + 4Y^3 X^3 \left(\frac{dy}{dx}\right) = 8X Y + 4X^2 \left(\frac{dy}{dx}\right) + 0$$

Step 2

Now arrange like terms and solve the equation

$$4Y^3 X^3 \left(\frac{dy}{dx}\right) - 4X^2 \left(\frac{dy}{dx}\right) = 8X Y - 3X^2 Y^4$$

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Step 3

Since there are two commonalities here, we can factorize

$$\left(\frac{dy}{dx}\right)(4Y^3X^3 - 4X^2) = 8XY - 3X^2Y^4$$

Step 4

Solve for $\left(\frac{dy}{dx}\right)$

$$\frac{\frac{dy}{dx}\cancel{(4Y^3X^3 - 4X^2)}}{\cancel{(4Y^3X^3 - 4X^2)}} = \frac{(8XY - 3X^2Y^4)}{(4Y^3X^3 - 4X^2)}$$

$$\frac{dy}{dx} = \frac{(8XY - 3X^2Y^4)}{(4Y^3X^3 - 4X^2)} \qquad \frac{dy}{dx} = \frac{(8)(1)(1) - 3(1)^2(1)^4}{(4)(1)^3(1)^3 - (4)(1)^2} = \frac{5}{0}$$

So the slope of the tangent line at (1,1) is 5

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Step 5

Find the equation of the tangent line at (1,1)

To find the equation, use the formular $y-y1 = m(x-x1)$

$y-y1 = m(x-x1)$

$y-1 = 5(x-1)$

$y-1 = 5x-5$

$y = 5x-5+1$

$y = 5x-4$

So the equation of the tangent line is $y = 5x-4$

By professor Ebou Janha 07-20-2026